GRAPH-WAVELET FILTERBANKS FOR EDGE-AWARE IMAGE PROCESSING

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ABSTRACT

In our recent work, we proposed the construction of critically-sampled wavelet filterbanks for analyzing functions defined on the vertices of arbitrary undirected graphs. These graph based functions, referred to as graph-signals, provide a flexible model for representing many datasets with arbitrary location and connectivity. An application area considered in this work is image-processing, where pixels can be connected with their neighbors to form undirected graphs. In this paper, we propose various graph-formulations of images, which capture both directionality and intrinsic edge-information. The proposed graph-wavelet filterbanks provide a sparse, edge-aware representation of image-signals. Our preliminary results in non-linear approximation and denoising using graphs show promising gains over standard separable wavelet filterbank designs.

Index Terms— Graph-wavelet filterbanks, edge-aware transforms, image compression, Image denoising

1. INTRODUCTION

While standard separable extensions of wavelet filterbanks to higher dimensional signals, such as 2-D images, provide useful multi-resolution analysis, they do not capture the intrinsic geometry of the images. For example, these extensions can capture only limited (mostly horizontal and vertical) directional information. This means, if the object boundaries in an image are neither horizontal nor vertical, e.g., diagonal or round shape, the resulting transform coefficients tend not to be sparse and high pass wavelet components can have significant energy. Therefore, more powerful representations are sought for images, in which basis functions can adapt to the directionality and edge-information contained in the image. Among the various solutions proposed, some transforms, such as 2-D Gabor wavelets [1] and complex wavelets [2], provide extra dimensionality at the cost of producing an over-sampled output. Other designs such as curvelets [3] and contourlet transforms [4], which provide a dictionary of anisotropic edge-aware basis functions, require higher complexity and suffer from the same problem of oversampling. Some other designs such as bandlets [5], directionlets [6] and tree-based lifting transforms [7] provide critically sampled transforms based on side-information about geometric flows in the image.

Images can also be viewed as graphs, by treating pixels as nodes, pixel intensities as graph-signals, and by connecting pixels with their neighbors in various ways. The advantage of formulating images as graphs is that different graphs can represent the same image, which offers flexibility of choosing the graphs that have useful properties. In particular, the weights of the links can be adjusted heterogeneously at each nodes to accommodate for the the edge-information present in the image. An example of weighted image-graph formulation is the anisotropic diffusion based image smoothing considered in [8]. In our recent work [9, 10], we designed two-channel wavelet-filterbanks for any undirected weighted graphs, with vertices (nodes) as data-sources. These filterbanks are critically sampled and provide basis elements which are localized in both spatial and frequency domain of the graph. Further, they can be implemented using an iterated separable filterbank structure, and thus provide a multi-resolution analysis of graph-signals. In [10], we applied these filterbanks to undirected unweighted graph representations of images, and showed that the interpretation of resulting graph-wavelets transforms is analogous to classical wavelet decompositions.

Our contribution in this work, is to formulate images as weighted graphs, which capture the underlying geometric structure of the image, and to apply our proposed graph-based wavelet-filterbanks [10] on these graphs. We provide preliminary results related to image non-linear approximation and denoising that show promising gains over standard separable wavelet transforms. The remainder of this paper is organized as follows: in Section 2, we first provide an overview of the critically sampled wavelet filterbanks on graphs, and propose different graph representations of the images based on the edge-information present in the image. In Section 3, we present the implementation details of our proposed design and compare its performance with standard wavelet decompositions. Finally we conclude the paper in Section 4.

2. SYSTEM DESCRIPTION

2.1. Overview of the graph-wavelet filterbanks

The graph-wavelet filterbanks proposed in our recent work [10], operate on any undirected weighted graph \( G = (V, E) \), where nodes in \( V \) are data-sources, connected to each other via a set of weighted links in \( E \). Data on the vertices of these graphs can be visualized as a finite collection of samples termed as graph-signals. The general idea of the proposed design [10] is to first decompose the underlying graph \( G \), into a set of \( K \) bipartite subgraphs \( B_i = (L_i, H_i, E_i) \) for \( i = 1, 2, \ldots, K \), using an iterative decomposition scheme. In this scheme, at each iteration stage \( i \), the bipartite subgraph \( B_i \) covers the same vertex set: \( L_i \cup H_i = V \), and \( E_i \) consists of all the links in \( E = \bigcup_{j=1}^{i-1} E_{j} \) that connect vertices in \( L_i \) to vertices in \( H_i \). Given such a decomposition, a two-channel wavelet filterbank is implemented in \( K \)-stages (“dimensions”), such that filtering and down-

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1. The frequency of graph is defined in terms of eigenvalues of the normalized graph Laplacian matrix.

2. A bipartite graph \( B_i = (L_i, H_i, E_i) \) is a graph whose vertices can be divided into two disjoint sets \( L_i \) and \( H_i \), such that every link connects a vertex in \( L_i \) to one in \( H_i \)
solutions which lead to small reconstruction errors and loss of orthogonality. However, they are not well-localized in the spatial domain and in practice, we compute polynomial approximations of exact solutions which lead to small reconstruction errors and loss of orthogonality.

2.2. Graph representation of images

Digital images are 2-D regular signals, but they can also be viewed as graphs by connecting every pixel (node) in an image with its neighboring pixels (nodes) and by interpreting pixel values as the values of the graph-signal at each node. Graph representations of the regularly sampled signals have been shown to be promising in practice recently [11, 12]. In our experiments, we use an 8-connected representation $G$ of an image as shown in Figure 2. In this representation, each pixel has two types of connections with its neighbors: (a) rectangular connections with NWSE neighbors, and (b) diagonal connections with its diagonal neighbors. Note that, adding more directions to the graph, for example, by linking each pixel with its 2-hop neighbors, is possible but is not considered in our present work. In the 8-connected image graph $G$, separating out rectangular and diagonal links into separate graphs leads to two bipartite subgraphs $B_1$ and $B_2$ as shown in Figure 2. The importance of each dimension can be changed by adjusting the weights of the links in each bipartite-subgraph. Given such a decomposition, we can implement a two-stage (“two-dimensional”) graph-wavelet filterbank, as described in [10], where the filtering operations in the first dimension capture the variations along rectangular directions and those in the second dimension capture the variations along the diagonal directions. The overall wavelet filterbank has 4 output channels, and the downsampling pattern in each channel is identical to a downsampling-by-4 pattern for standard separable case. The nodes sampled in different channels are shown by different colors in the 8-connected graph $G$ in Figure 2.

2.2.1. Edge-aware graph representations

Graph representation of images provide a simple way to accommodate the edge-information present in the images, by adjusting the weights of the pixels near the edges. In this approach, first the pixels at the edges (i.e., pixel whose intensities change sharply from their neighboring pixels), are detected using standard edge-detection algorithms. Subsequently, the links between each edge-pixel and its neighbors are tagged as either regular or less-reliable, depending on the difference between pixel intensities across the link being low or high, respectively. Then in the edge-aware graph representation, the less-reliable links are either completely removed or assigned a lower link-weight than the regular links. Similar constructions have been proposed in recent work [12, 13], but these constructions use lifting transforms and block transforms respectively, and do not use graph-filterbanks. In our proposed design, we choose to assign a lower link-weight for less-reliable links, as completely removing links around edge-pixels sometimes create isolated pixels (holes) in the graph, which do not participate in computing the wavelet transform, and thus need to be separately accounted for. Consequently, the graph-wavelet filters on the resulting weighted graph have most of their energy on one side of the edge, and produce less number of non-zero wavelet coefficients at the edges than in the case of unweighted image graphs. This is demonstrated by an example in Figures 3 and 4.

2.2.2. Downsampling image graphs

Similar to standard wavelet transforms, the graph-wavelet filterbanks can be recursively applied at multiple levels, treating the LL channel output coefficients to be the new graph-signal, operating upon the...
downsampled graph constructing using the LL channel nodes only. In the proposed 8-connected image graph representation, since the LL channel nodes are uniformly sampled, the downsampled graph using LL nodes is made 8-connected by connecting each LL pixel to its neighboring 8 LL pixels. Further, the link-weights in the downsampled graph represent sum of the weights of the paths connecting LL nodes in the original graph, along each direction (vertical, horizontal, diagonal and off-diagonal), where the weight of the path is product of the weights of the links that it consists of.

3. EXPERIMENTS

In our experiments, we choose an undirected 8-connected representation of images as described in the Section 2.2. For edge-detection in an image, we use standard Gaussian filtering followed by thresholding. In addition, we perform a connected component analysis to weed out small clusters of edge-pixels (of size less than 200), and dilate the remaining edges using a $2 \times 2$ structuring element to fill out the empty corners in the edges. For each edge-pixel the links between the pixel and its 8 neighbors are divided into two sets by applying a two-class clustering, based on the intensity difference. The links in the cluster with high intensity difference are declared less-reliable and their weights are adjusted to one-fourth of the weights of regular links (which is set to 1). The resulting graph has a binary weight distribution of links (regular/less-reliable). The graphs in the subsequent levels of decompositions are generated from downsampling the first level graph as described in the Section 2.2.2, and have a more varied link-weight distribution. The graph at each level of decomposition is further decomposed into a rectangular-link only and a diagonal-link only bipartite graph as shown in Figure 2, and a two-stage two-channel graph-QMF filterbank [10] is then applied at each level. The filters in the filterbank are chosen to be polynomial approximations of graph-QMF filters proposed in [10] with parameter $m = 30$ (for $m^{th}$ order of approximation). The wavelet coefficients can be computed iteratively at each level, using $m$ one-hop localized operations at each node. Thus, the computational complexity of graph filterbanks for an $N$-pixel image is $O(mN)$.

3.1. Image non-linear approximation

We now compare the proposed graph-based filterbanks with existing CDF 9/7 filters used in JPEG2000, using non-linear approximation with $k$-largest wavelet coefficients. Figure 5, shows PSNR and SSIM [14] values plotted against fraction of detail coefficients used in the reconstruction of Lena ($512 \times 512$) image. It can be seen from both the plots that graph-QMF filterbanks achieve better compression than the standard CDF 9/7 filterbanks. This is because the graph-QMF filterbanks capture a more circular variation of the image-signal than the separable case. Among the graph-based wavelet transforms, the edge-weighted formulations perform better than unweighted formulation. This makes sense, as the weighted graph-formulations of the image are edge-aware and produce fewer wavelet coefficients compared to unweighted graphs near the edges. However, the performance gain does not include additional edge-map information which can eclipse the gain. Recent work [11, 13] using transforms based on similar edge-map information have shown that this trade-off is favorable. Formulating the trade-off between extra performance gain using edge-weighted graphs and the edge-information needed, as an optimization problem constitutes part of our ongoing work. Figure 6 shows the reconstructed image with largest 1% detail coefficients in all described cases. It can be seen that perceptually the reconstructed images using both graph-QMF wavelets look sharper than the reconstruction with standard CDF 9/7 wavelet reconstruction. However, the reconstructions using the unweighted graph-QMF wavelets have ringing artifacts near some edges, which disappear when we use the edge-weighted graph formulation. Thus, the edge-weighted graphs and corresponding graph-wavelet filterbanks produce a sparser representation of edges, than the standard separable wavelets.

3.2. Image denoising

The proposed graph-based filterbanks can also be applied on wavelet based image de-noising. In our implementation, we adopt the denoising scheme proposed in [15], which at first decompose the image into different frequency subband, and apply local soft thresholding on each coefficient in each subband respectively. The threshold value $T_x$ for each wavelet coefficient can be determined as:

$$T_x = \frac{\sigma_x^2}{\sigma_n}$$  \hspace{1cm} (1)

where $\sigma_x$ is the standard deviation of the coefficient considered, and $\sigma_n$ is the standard deviation of noise. $\sigma_x$ can be estimated using
context modeling[15], and the $\sigma_n$ can be estimated from the HH subband using the robust median estimator. Using this de-noising algorithm, we then apply our proposed graph filterbank on images with different noise levels and compare the restoration performance with the results from wavelet CDF 9/7. As a proof of concept, we use the non-noisy image (which is not available in real-world scenario) to detect edge-pixels, for computing edge-aware graph representation. However, the performance degrades when edge-detection is performed on the denoised images because of the generation of spurious reliable links in the graph, and remains an area of our ongoing research work. Fig. 7 shows both the comparisons of PSNR and structural similarity (SSIM) index with different noise levels. It can be seen that the proposed graph-QMF filterbank outperform almost all other methods under both evaluation criteria. The only exception is for noise level $\sigma = 5$, which results mainly from the imperfect reconstruction of polynomial approximated filterbank. For higher noise level, the advantage of edge-awareness in graph filterbank is better illustrated.

4. CONCLUSION AND ONGOING WORK

In this paper, we have proposed a novel method of processing 2D images using a graph-based wavelet filterbank design. The graph-based wavelet filterbanks designed in our recent work [10] can be applied to the vertices of any arbitrary undirected graph. We have proposed a graph representation of images in which pixels are connected with their neighbors to form undirected graphs. The graph formulation captures the geometric structure of the image by linking pixels in different directions and by adjusting the weights of the links near edges. Preliminary results show gains in the image non-linear approximation and denoising application over standard wavelet filterbank. Our ongoing work includes studying a more heterogeneous distribution of link weights in the image-graphs and its impact on the image-formulation.

5. REFERENCES